Analytical Solution of Radiated Acoustic Field by Moving Monopolar and Dipolar Sources

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***Abstract*—It is challenging to predict the patterns of radiated acoustic field from a complex moving source while this topic is important in the field of ocean acoustics. A possible method is to decompose the complex field into the addition of fundamental sources such as monopolar and dipolar ones. In this study, we derive the analytical solution of radiated acoustic fields generated by a moving monopolar or dipolar source, and the combination of them based on the Lorentz transformation. The analytical expressions of the radiated sound pressure by a moving monopolar or dipolar source are given with no limit on the moving direction and excitation frequency. Numerical simulations are conducted to reveal the Doppler effect of the moving sources and the potential application of the combination of monopolar and dipolar sources to simulate complex fields. This work may help design acoustic models to predict the radiated sound field by complex sources in the field of ocean engineering.**

Keywords—Lorentz transformation, Radiated acoustic field, moving target, monopolar and dipolar sources, Doppler effect, Marine Engineering

I. INTRODUCTION

The objective of target motion analysis is to ascertain the motion characteristics of unidentified targets in order to achieve precise target positioning and tracking. Conventional techniques for analyzing and positioning underwater targets typically involve the use of active localization methods. These methods involve the deployment of active sources that emit directed beams towards underwater targets, enabling the extraction of motion data from the reflected signals. Although this method offers high accuracy and practicality, it is hindered by the drawback of diminished stealth capabilities due to the requirement of emitting powerful sound fields for effective functionality.

Passive acoustic detection methods offer an alternative approach by utilizing the noise emissions of underwater targets to extract essential information such as their location and movement. This technique has garnered considerable interest from researchers on a national and international scale due to its improved stealth capabilities and practicality. It is important to note that the movement of targets influences the modulation of the acoustic field radiated and scattered. Hickling and Robert initially conducted calculations on sound scattering by a movable rigid sphere in an ideal medium [1], with Temkin and Leung later expanding this to include viscous fluids [2]. Olsson utilized the null-field approach and T-matrix method to present findings on acoustic scattering by a movable rigid body in a fluid [3]. Venkov developed a formula accounting for the impact of Lorentz transformations on scattering amplitude and analyzed the scattering of plane acoustic waves by a moving soft sphere in open space [4]. Ma introduced an analytical time-domain formulation based on the Ffowcs Williams-Hawkings (FW-H) equation for predicting the acoustic velocity field generated by moving objects [5]. More recently, Roux et al. investigated the acoustic radiation force exerted on a moving monopolar source, demonstrating that the asymmetry of the emitted field due to the Doppler effect results in a radiation force opposing the source's motion [6]. They also showed that a translating dipolar acoustic source experiences a self-induced radiation force opposing its motion [7].

The research presented above indicates that the scattering and radiated acoustic field of a target in motion exhibit greater complexity in comparison to those of stationary targets. The acoustic fields are influenced by the relative movement of sources, offering potential for applications in source localization and identification. This study develops equations utilizing Lorentz transformation to determine the radiated acoustic field of a single moving monopolar and dipolar source. Through the application of the superposition principle to acoustic fields, the intricate acoustic field produced by moving targets can be replicated by combining moving monopolar and dipolar sources with specific weights and parameters.

II. WAVEFIELD RADIATED BY A MOVING MONOPOLAR SOURCE

When a time-varying source Q(t) is situated at a particular location within a fluid medium, the point mass source can be considered analogous to a uniformly pulsating sphere with an infinite radius. This analogy is based on the concept that the influx and efflux of mass can be compared to the expansion and contraction of the sphere, which serves as a monopole source.

A. Wave equation by a moving monopolar source

The pressure is expressed as $P = P_0 + p$, with P_0 denoting the static pressure and p representing the sound pressure. Density is denoted as $\rho = \rho_0 + \rho'$, where ρ_0 stands for the static density and ρ' represents the density perturbation. For simplicity, higher-order quantities are disregarded, and the fluid is assumed to be inviscid and homogeneous. Consequently, this assumption facilitates the derivation of the motion equation:

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$$\rho_0 \frac{\partial \boldsymbol{v}}{\partial t} = -\nabla p \quad (1)$$

where **v** is the acoustic velocity. Similarly, the continuity equation can be written as:

$$-\rho_0 \nabla \cdot \boldsymbol{v} = \frac{\partial \rho'}{\partial t} \quad (2)$$

The equation of state is given with the relation to the sound speed c_0:

$$p = c_0^{\ 2} \rho' \quad (3)$$

Baes on (1), (2) and (3), the wave equation can be easily obtained:

$$\Delta p = \frac{1}{c_0^{\ 2}} \frac{\partial^2 p}{\partial t^2} \quad (4)$$

To simplify subsequent calculations, it is convenient to introduce the velocity potential denoted by ψ, defined as $\mathbf{v} = -(1/\rho_0)\nabla \psi$ so that $p = \partial \psi / \partial t$, and then the wave equation can be expressed in terms of ψ:

$$\Delta \psi = \frac{1}{c_0^{\ 2}} \frac{\partial^2 \psi}{\partial t^2} \quad (5)$$

Now, let us examine the activation of a monopolar sound emitter within the context of the wave equation. This monopolar emitter can be viewed as a mass emitter, with its strength dictated by the instantaneous mass flow rate $q(t)$ produced by the emitter. Subsequently, the source function $q(t)\delta(r\text{-}r_0)$ will be integrated on the right-hand side of the wave equation, where $\delta(r\text{-}r_0)$ represents the delta function. Therefore, for a monopolar emitter traveling at a velocity $U = Mc_0\boldsymbol{x}$ along a stationary x-axis (where M is the Mach number), the wave equation can be formulated as:

$$\Delta \psi - \frac{1}{c_0^{\ 2}} \frac{\partial^2 \psi}{\partial t^2} = -q(t)\delta(x - Mc_0 t)\delta(y)\delta(z) \quad (6)$$

B. Resolution of the wave equation

Initially, it is important to examine the scenario of a stationary monopolar source with a moment of M equal to zero. In accordance with acoustic principles, when dealing with a simple harmonic motion, the resolution to the wave equation is as follows:

$$\psi(r,t) = \frac{q(t \pm r / c_0)}{4\pi r} \quad (7)$$

Where $r = \sqrt{x^2 + y^2 + z^2}$ is the radial distance. The investigation now shifts towards examining the resolution of the wave equation in scenarios involving a mobile monopolar source. In such instances, the application of the Lorentz transformation method, a fundamental aspect of special relativity, becomes pertinent. Let us define $\gamma = 1/\sqrt{1-M^2}$ as the 'Lorentz acoustic boost'. The Lorentz transformation can be expressed as:

$$\begin{cases} x' = \gamma(x - Mc_0 t) \\ y' = y \\ z' = z \\ c_0 t' = \gamma(c_0 t - Mx) \end{cases} \quad (8)$$

And Eq. (6) becomes:

$$\Delta' \psi - \frac{1}{c_0^{\ 2}} \frac{\partial^2 \psi}{\partial t'^2} = -q(\gamma(t' + Mx'/c_0))\delta(x'/\gamma)\delta(y')\delta(z') \quad (9)$$

Given that the right side of Eq. (9) is null when $x' \neq 0$ for all t' based on the feature of the delta function. Using $\delta(x'/\gamma) = \gamma\delta(x')$, we can obtain:

$$\Delta' \psi - \frac{1}{c_0^{\ 2}} \frac{\partial^2 \psi}{\partial t'^2} = -\gamma q(\gamma t')\delta(x')\delta(y')\delta(z') \quad (10)$$

Here, we introduce a second set of variables:

$$\begin{cases} x'' = \gamma x' \\ y'' = \gamma y' \\ z'' = \gamma z' \\ c_0 t'' = \gamma c_0 t' \end{cases} \quad (11)$$

Subsequently, the wave equation can be transformed into:

$$\Delta'' \psi - \frac{1}{c_0^{\ 2}} \frac{\partial^2 \psi}{\partial t''^2} = -\gamma^2 q(t'')\delta(x'')\delta(y'')\delta(z'') \quad (12)$$

It is apparent that following the substitution of variables, the wave equation governing a monopole source in motion closely resembles that of a stationary source. As a result, the solution to equation (12) can be determined:

$$\psi(r'', t'') = \gamma^2 \frac{q(t'' \pm r''/c_0)}{4\pi r''} \quad (13)$$

And now, we need to revert the solution's form back to the original function of (x, y, z, t). The first step is in terms of (r', t') :

$$\psi(r', t') = \gamma^2 \frac{q(\gamma(t' \pm r'/c_0))}{4\pi \gamma r'} \quad (14)$$

with

$$\gamma(t' \pm \frac{r'}{c_0}) = \frac{\gamma}{c_0}\left[\gamma(c_0 t - Mx) \pm \sqrt{\left[\gamma(x - Mc_0 t)\right]^2 + y^2 + z^2}\right] \quad (15)$$

where $\gamma = 1/\sqrt{1-M^2}$, we obtain:

$$\gamma(t' \pm \frac{r'}{c_0}) = t - \frac{M(x - Mc_0 t) \pm \sqrt{(x - Mc_0 t)^2 + (y^2 + z^2)(1 - M^2)}}{c_0(1 - M^2)} \quad (16)$$

Hence, the solution for the mobile monopolar source is derived. Furthermore, the distance R, denoting the separation between the source and the point of observation, can be specified ($M < 1$):

$$R = \frac{M(x - Mc_0 t) + R_1}{1 - M^2} \tag{17}$$

with

$$R_1 = \sqrt{(x - Mc_0 t)^2 + (y^2 + z^2)(1 - M^2)} \tag{18}$$

Then the final solution becomes:

$$\psi(r,t) = \frac{q(t - R/c_0)}{4\pi R_1} \tag{19}$$

The pressure field then can be expressed as:

$$p = \frac{\partial \psi}{\partial t} = \frac{(1 - 1/c_0\, \partial R/\partial t) q'(t - R/c_0)}{4\pi R_1} - \frac{q(t - R/c_0)\, \partial R_1/\partial t}{4\pi R_1^{\,2}} \tag{20}$$

with the derivate $q' = dq/dt$.

III. WAVEFIELD RADIATED BY A MOVING DIPOLAR SOURCE

When a force that varies with time, denoted as $\mathbf{f}(t)$, is applied to a specific point in a fluid medium, it can be likened to an acoustic dipole. The disturbances on each side of the vibrating sphere demonstrate contrasting phases, leading to the emission of dipolar radiation.

A. Wave equation by a moving dipolar source

Consider a dipolar source moving at a velocity $U = Mc_0\boldsymbol{x}$ along a fixed axis $\boldsymbol{x}$. Define the force $\mathbf{f}(\mathbf{r}, t)$:

$$\mathbf{f}(\mathbf{r},t) = \mathbf{f}(t)\delta(x - Mc_0 t)\delta(y)\delta(z) \tag{21}$$

The force $\mathbf{f}(t)$ is assumed to be arbitrary for generality, allowing $\mathbf{f}(t)$ to be expressed as:

$$\mathbf{f}(t) = F\sin(\omega t)(x, y, z)^{\mathrm{T}} \tag{22}$$

with $(x, y, z)^{\mathrm{T}}$ a unit vector, F the force amplitude and ω the angular frequency.

As the essence of the dipolar source lies in the vibration of the fluid medium, we introduce the displacement $\mathbf{u}$ such that $\mathbf{v} = \partial\mathbf{u}/\partial t$. Just as with the scenario involving a mobile monopolar source, we can establish the wave equation for a dipolar source in motion:

$$c_0^{\,2}\Delta\mathbf{u} - \frac{\partial^2\mathbf{u}}{\partial t^2} = -\frac{1}{\rho_0}\mathbf{f}(t)\delta(x - Mc_0 t)\delta(y)\delta(z) \tag{23}$$

With the Lorentz transformation of Eq. (8), given that the right side is null when $x' \neq 0$ for all t', and using $\delta(x'/\gamma) = \gamma\delta(x')$, the equation can be rewritten:

$$c_0^{\,2}\Delta'\mathbf{u} - \frac{\partial^2\mathbf{u}}{\partial t'^2} = -\frac{\gamma}{\rho_0}\mathbf{f}(\gamma t')\delta(x')\delta(y')\delta(z') \tag{24}$$

By employing the same substitution of variables as Eq. (11), the final wave equation transforms into:

$$\Delta''\mathbf{u} - \frac{1}{c_0^{\,2}}\frac{\partial^2\mathbf{u}}{\partial t''^2} = -\frac{\gamma^2}{c_0^{\,2}\rho_0}\mathbf{f}(t'')\delta(x'')\delta(y'')\delta(z'') \tag{25}$$

B. Resolution of the wave equation

The wave equation for a moving dipolar source bears resemblance to that of a moving monopolar source, with the exception of the coefficient component. Consequently, the solution for the former can be easily derived:

$$\mathbf{u} = \frac{\gamma^2}{c_0^{\,2}\rho_0}\frac{\mathbf{f}(t'' \pm r''/c_0)}{4\pi r''} \tag{26}$$

After establishing the distance R between the source and the observation point in the context of solving for the moving monopole source, the ultimate solution can be articulated by utilizing the original variables via a similar process of back substitution:

$$\mathbf{u} = \frac{\mathbf{f}(t - R/c_0)}{4\pi c_0^{\,2}\rho_0 R_1} \tag{27}$$

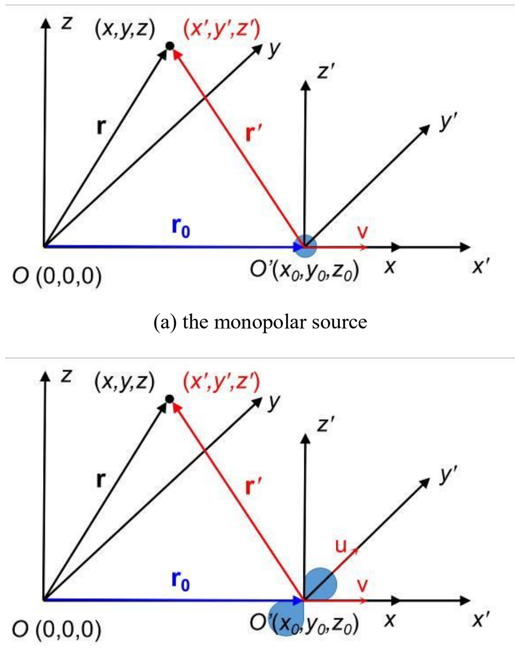
where R and R_1 are the same as Eqs. (17) and (18).

The pressure field can be represented by the relationship between the displacement $\mathbf{u}$ and the pressure p, the pressure field then can be expressed as:

$$p = -c_0^{\,2}\rho_0\nabla\cdot\mathbf{u} = -\frac{1}{4\pi}\nabla\cdot\frac{\mathbf{f}(t - R/c_0)}{R_1} \tag{28}$$

IV. STIMULATION OF ARBITRARY COMPLEX WAVEFIELD

Having derived the mathematical equations for the sound pressure produced by a mobile monopolar source and a mobile dipolar source along a stationary axis $\boldsymbol{x}$, we can construct a visual representation depicting the overall and specific moving coordinates of both sources, as depicted in Fig. 1.



(a) the monopolar source

(b) the dipolar source

Fig. 1. Global and local moving coordinates

The acoustic field distribution in various directions and speeds for specific sources can be determined. By applying the superposition principle of acoustic fields, a combination of monopolar and dipolar sources can be utilized as fundamental sources to replicate the complex acoustic field produced by a moving target. To improve the visualization of the acoustic field, simulations are conducted in a two-dimensional acoustic field. Assuming the speed of sound underwater is 1500 m/s, although typical underwater vehicles move at speeds below 100 m/s, simulations are performed with speeds up to 500 m/s to better illustrate the Doppler effect of the radiated field caused by spatial motion. These simulations are carried out using Matlab R2023b.

A. A single moving monopolar source

Initially, we investigate the Doppler shift induced by a monopolar source in motion at a consistent speed, analyzed at varying time intervals. Specifically, we consider a monopolar source in motion along the x-axis with a frequency of 5000 Hz and a velocity of 300 m/s. The acoustic pressure distributions at time instances of 0.0001s, 0.0002s, and 0.0003s are depicted in Fig. 2.

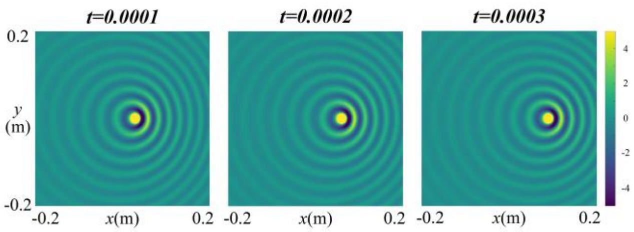


Fig. 2. Radiated acoustic field by a moving monopolar source with the speed v=300 m/s in the x-axis direction at the frequency of 5000 Hz for different time.

Furthermore, Fig. 3 illustrates the pressure distributions of a monopolar source with varying velocities concurrently. Assuming the monopolar source oscillates at a frequency of 5000 Hz and travels along the x-axis, simulations were conducted to analyze the resulting acoustic field distributions when the velocities of motion are 100 m/s, 300 m/s, and 500 m/s, each evaluated at the time instant of 0.0002 seconds.

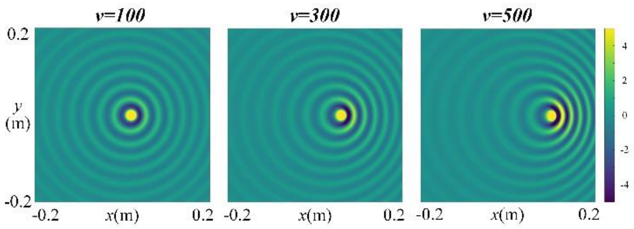


Fig. 3. Radiated acoustic field by a moving monopolar source at different moving speed in the x-axis direction. The time is fixed as t=0.0002s.

As illustrated in Fig. 3, it is apparent that an increase in the velocity of the monopolar source results in a higher frequency of the radiated field detected by the observation point in the forward direction, as indicated on the right side of the field. Conversely, the frequency perceived by the observation point situated behind the source decreases, as shown on the left side. This observation aligns with the principles of the Doppler effect. Nevertheless, when the velocity of the object is relatively lower, such as at 100 m/s, the manifestation of the Doppler effect is less pronounced.

B. A single moving dipolar source

In a similar manner, the acoustic field distribution is computed at various time points (t=0.0001s, 0.0002s, 0.0003s) and speeds (v=100m/s, 300m/s, 500m/s) as the dipolar source moves along the axis $\boldsymbol{x}$ with a frequency of 5000. The orientation of the dipolar source's vibration can vary. This study focuses on two distinct scenarios: one where the oscillation direction aligns with the motion direction, and another where it is perpendicular to the motion direction.

First, we show the spatial pressure patterns where the oscillation direction is consistent with the direction of motion, as shown in Figs. 4 and 5.

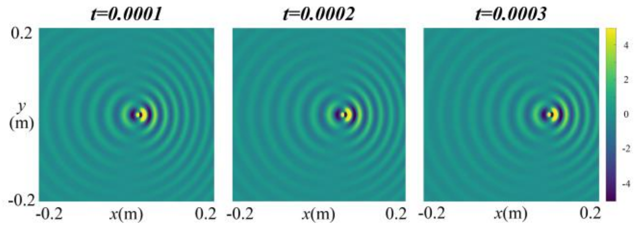


Fig. 4. Radiated acoustic field by a moving dipolar source with the speed v=300 m/s in the x-axis direction source. The oscillation direction is consistent with the moving direction.

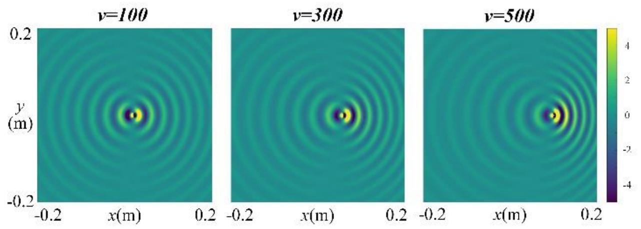


Fig. 5. Radiated acoustic field by a moving dipolar source at different speed in the x-axis direction. The time is fixed as t=0.0002s and The oscillation direction is consistent with the moving direction.

In addition, below in Figs. 6 and 7 are the simulation results when the oscillation direction is perpendicular to the direction of motion.

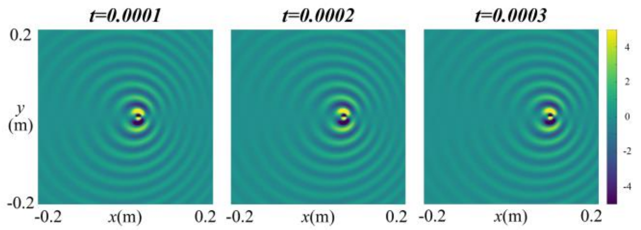


Fig. 6. Same as Fig. 4 except that the vibration of the dipolar source is in the y-axis direction which is perpendicular to the direction of motion.

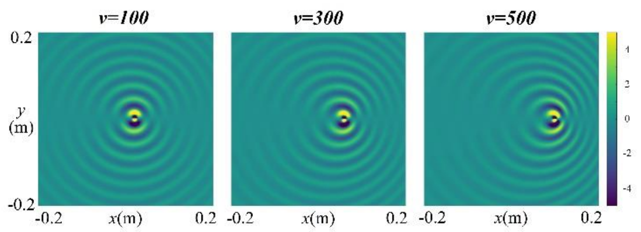


Fig. 7. Same as Fig. 5 except that the vibration of the dipolar source is in the y-axis direction which is perpendicular to the direction of motion.

An analysis of the model indicates that the directional nature of the dipolar sound source results in noticeable changes in the sound field due to the Doppler effect at speeds of 300 m/s and 500 m/s. However, akin to the scenario with a monopolar sound source, the Doppler effect is minimal at lower speeds (v=100 m/s).

C. *Combination of monopolar and dipolar sources*

Thus far, we have developed models for simulating the prediction of pressure fields emitted by monopolar and dipolar sound sources at various speeds, excitation frequencies, and directions. By leveraging these models, we can employ the concept of acoustic field superposition to combine these two types of sound sources and simulate intricate acoustic fields resulting from the movement of targets.

Initially, we examine a scenario in which a monopolar source and a dipolar source operate at a common frequency of 5000 Hz. It is assumed that two monopolar sources are in motion along the x-axis in opposite directions, while a dipolar source moves along the negative y-axis, with its vibration direction aligned with its motion. The resulting pressure field pattern resembles that of a bear, as illustrated in Fig. 8.

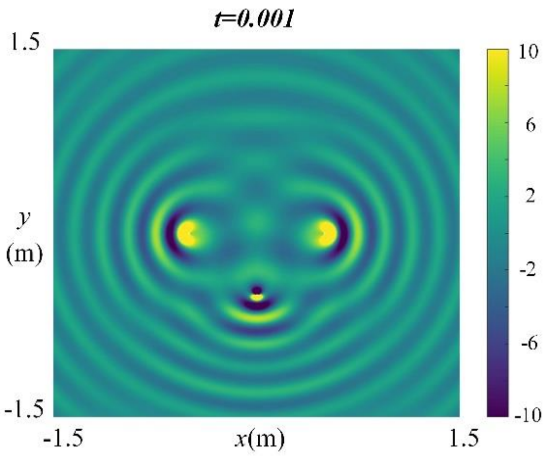


Fig. 8. Complex acoustic field by the addition of two monopolar and one dipolar sources at the same frequency 5000 Hz.

Next, we examine the occurrence of the dipolar source having a frequency double that of the monopolar source at 10000 Hz. Two monopolar sources are observed to move along the x-axis in opposite directions, while the dipolar source moves along the y-axis in a positive direction. Furthermore, the dipolar source oscillates in alignment with its motion. The pressure field is illustrated in Fig. 9.

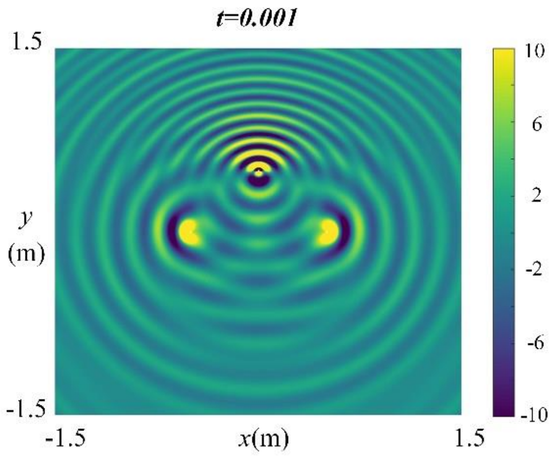


Fig. 9. Complex acoustic field two monopolar and one dipolar sources at different frequencies. The frequency for the monopolar sources is 5000 Hz, while for the dipolar source is 10000 Hz.

In brief, the integration of mobile monopolar and dipolar sources enables the simulation of radiated acoustic fields from intricate targets. Parameters such as initial position, motion direction, velocity, frequency, and others for both monopolar and dipolar sources can be adjusted accordingly, offering a versatile numerical tool with substantial adaptability and comprehensiveness for simulating mobile complex targets.

V. Conclusion

In this paper, we aim to replicate the intricate acoustic field produced by mobile targets using monopolar and dipolar sources. Initially, the wave equation for a moving monopolar and dipolar source is derived via the Lorentz transformation. Subsequently, the wave equation is solved through parameter substitution, facilitating the derivation of analytical expressions for the radiated acoustic field of the moving monopolar and dipolar sources.

Following this, simulations are conducted on the spatial distribution of the acoustic field emitted by the moving monopolar and dipolar sources. The numerical simulations of acoustic field patterns at different times for a constant speed reveal significant Doppler effect for both monopolar and dipolar sources at higher speeds. Observing the acoustic field patterns at various speeds simultaneously indicates that, for a fixed source frequency, the Doppler frequency shift induced by the target's motion is pronounced at high speeds but weak at lower speeds.

Subsequently, by combining multiple monopolar and dipolar moving sources without constraints on their moving direction, speed, and excitation frequency, complex acoustic fields can be generated as anticipated. In the subsequent phase, efforts will be directed towards solving the inverse problem of characterizing a moving complex acoustic field by amalgamating several monopolar and dipolar sources. This research has the potential to aid in the development of acoustic models for ship radiated noise [8], directional transducer arrays [9], and underwater vehicle clusters [10].

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